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(12) INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(19) World Intellectual Property Organization
International Bureau

(43) International Publication Date
5 December 2002 (05.12.2002)

PCT

(10) International Publication Number
WO 02/097656 A2

(51) International Patent Classification[7]: G06F 17/10

(21) International Application Number: PCT/CA02/00696

(22) International Filing Date: 10 May 2002 (10.05.2002)

(25) Filing Language: English

(26) Publication Language: English

(30) Priority Data:
2,349,041 28 May 2001 (28.05.2001) CA

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(81) Designated States (national): AE, AG, AL, AM, AT, AU, AZ, BA, BB, BG, BR, BY, BZ, CA, CH, CN, CO, CR, CU, CZ, DE, DK, DM, DZ, EC, EE, ES, FI, GB, GD, GE, GH, GM, HR, HU, ID, IL, IN, IS, JP, KE, KG, KP, KR, KZ, LC, LK, LR, LS, LT, LU, LV, MA, MD, MG, MK, MN, MW, MX, MZ, NO, NZ, OM, PH, PL, PT, RO, RU, SD, SE, SG, SI, SK, SL, TJ, TM, TN, TR, TT, TZ, UA, UG, US, UZ, VN, YU, ZA, ZM, ZW.

(84) Designated States (regional): ARIPO patent (GH, GM, KE, LS, MW, MZ, SD, SL, SZ, TZ, UG, ZM, ZW), Eurasian patent (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), European patent (AT, BE, CH, CY, DE, DK, ES, FI, FR, GB, GR, IE, IT, LU, MC, NL, PT, SE, TR), OAPI patent (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML, MR, NE, SN, TD, TG).

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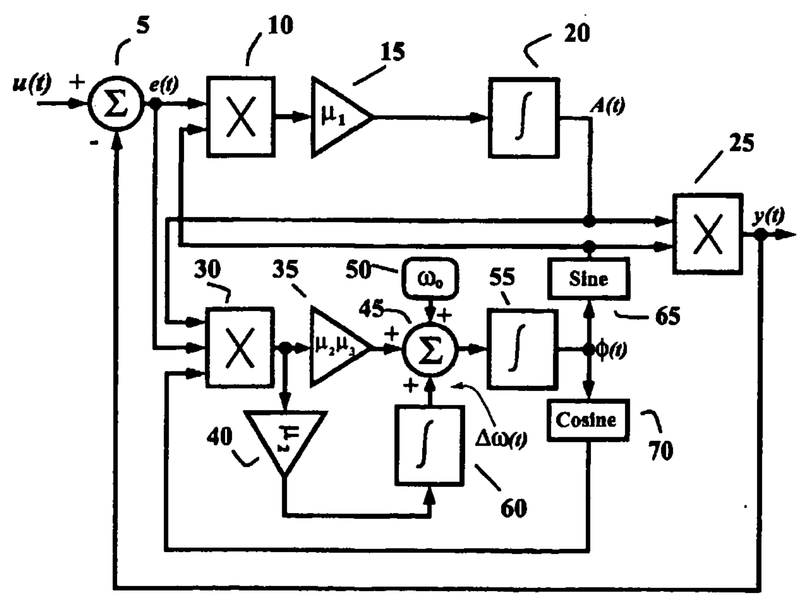
— *of inventorship (Rule 4.17(iv)) for US only*

Published:
— *without international search report and to be republished upon receipt of that report*

[Continued on next page]

(54) Title: SYSTEM AND METHOD OF EXTRACTION OF NONSTATIONARY SINUSOIDS



(57) Abstract: A signal processing method and an apparatus built on the basis of such a method are disclosed. The present invention provides a means of extraction of a more or less specified single sinusoidal component of a given nonstationary signal, which may be polluted by noise and undesired components, and tracking variations of the amplitude, phase and frequency of such a sinusoid over time. It also directly provides the estimates of potentially time-varying parameters of the sinusoidal component of interest such as its amplitude, frequency and phase. The signal analysis/synthesis tool of the invention, when used as a complete system in its own or as the fundamental building block of single-core or multi-core systems, finds applications in diverse areas of engineering and science.

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SYSTEM AND METHOD OF EXTRACTION OF NONSTATIONARY SINUSOIDS

TECHNICAL FIELD

The present invention relates to signal processing techniques and systems and, in particular, to a method of extraction of sinusoidal signals of time-varying nature and an apparatus built on the basis of such a method, which is capable of extracting a specified single sinusoidal component of an input signal, potentially containing other components and noise, and tracking variations of the amplitude, phase and frequency of such a sinusoid over time. Such a signal analysis/synthesis tool, used in isolation or as the fundamental building block of single-core or multi-core systems, finds applications in diverse areas of engineering and science, ranging from signal detection, extraction, measurement and synthesis in engineering to time-frequency analysis of mechanically generated signals in science. Some examples of its applications are the detection of dual-tone multi-frequency (DTMF) signals in telephony, signal recovery in noisy biopotential signals, and extraction and measurement of power line signals.

BACKGROUND ART

The effect of periodic phenomena, registered as signals, is usually studied using common signal analysis tools such as Fourier analysis. A given signal, as long as it manifests some periodicity, can be thought of as being composed of a series of single sinusoidal components or, more commonly named, tones. Frequency domain characterization of signals amounts to the identification of individual constituting components of a given signal and their corresponding values of amplitude and phase. In the case of nonstationary signals, all characteristics of the constituting components, including frequency, may vary over time. The main shortcoming of Fourier-based methods is their inherent fixed-frequency assumption which limits their applicability to the majority of real signals, the frequency characteristics of which may vary with time. There have been numerous attempts to devise adaptation mechanisms to be incorporated into signal analysis tools to render them useful for analysis of quasi-periodic signals. Linear adaptive filtering is an example of such methods reported so far with its success and shortcomings.

Aside from the time-varying quality of real signals, very often signals are buried under noise and disturbances and may thus be severely distorted. Often, a useful signal analysis tool loses its efficiency when it is applied to signals affected by noise and disturbances. Thus, it is often necessary to recover the signal itself out of the background noise, especially under nonstationary conditions. Extraction of signal itself, and not just its characteristics, is of particular interest in applications where synchronization matters; i.e. where the total phase information of signal is important. In such cases, a single sinusoidal component of a given signal, or the totality of a number of such sinusoids, is to be extracted, or equivalently a desirable noise-free synchronized signal is to be synthesized. Synthesization of signals synchronous with a given reference signal finds applications in more areas than those dealing with extraction of signals out of noise and may very well include those applications in which phase-locked loop (PLL) circuits and systems are employed.

Considering the inadequacy of the performance of the available standard signal analysis tools, such as Fourier-based techniques, adaptive filters and PLLs, in extracting sinusoids of time-varying nature buried under noise in a unified way, it is not difficult to explain the existing diversity of the methods, each designed to tackle a specific type of problem. For example, the following United States of America patents describe inventions, each aiming at extraction of some information of sinusoids under more or less defined conditions:

6175818	Jan. 16, 2001	King,
6122652	Sep. 19, 2000	Jin et al.,
6088403	Jul. 11, 2000	Johnson,
6006083	Dec. 21, 1999	Tong et al.,
5734577	Mar. 31, 1998	Chesir, et al.,
5721689	Feb. 24, 1998	Hart, et al.,
5696578	Dec. 09, 1997	Ishida et al.,
5583785	Dec. 10, 1996	Hainey,
5583784	Dec. 10, 1996	Kapust, et al.

DISCLOSURE OF INVENTION

The present invention offers a means of extraction and estimation of the parameters of individual constituting sinusoids present in a given signal alternative to standard Fourier transform or linear adaptive filtering as regards signal analysis, and alternative to phase-locked loops as regards signal synthesis while maintaining a structural simplicity and high speed of convergence both comparable with those of Fourier-based techniques. Unlike Fourier-based techniques, once the individual constituting components of a given signal are extracted, time variations in the characteristics of the components are registered and tracked.

It is thus an object of the present invention to provide a means of extraction of a more or less specified desired sinusoidal component of a given signal, characteristics of which such as its amplitude, phase and frequency may vary with time.

Accordingly, the present invention offers a means of synthesization of a signal synchronous with a desired component of a given reference time-varying signal, thereby rendering it a signal synthesis tool.

It is another object of the present invention to provide a means of estimation of time-varying parameters of a more or less specified sinusoidal component of a given signal such as its amplitude, constant and total phase, and frequency.

Accordingly, the present invention offers a means of detection and measurement of characteristics of constituting components of a given signal, thereby rendering it a time-frequency analysis tool.

It is also another object of the present invention to provide a means of extraction of a more or less specified sinusoidal component of a given signal which may be highly polluted by noise and may be distorted by various external disturbances.

Advantageously, the signal synthesis tool offered by the embodiments of the present invention provides a noise-free signal synchronous with a desired component of a given

time-varying signal which may be polluted by noise, thus featuring it as a noise elimination/reduction technique.

Also advantageously, the time-frequency analysis tool offered by the embodiments of the present invention has a high degree of noise immunity, thus featuring it as a tool suitable for analysis of intensely noisy time-varying signals.

It is yet another object of the present invention to maintain structural simplicity so that the computational needs are comparable with those of simple Fourier-based techniques. In this respect, the signal analysis/synthesis tool offered by the embodiments of the present invention is particularly advantageous as regards its simplicity of structure.

Finally, it is another object of the present invention to present an algorithm, efficiency of which in terms of convergence time is comparable with that of Fourier-based techniques. In this respect, the signal analysis/synthesis tool offered by the embodiments of the present invention is particularly advantageous as regards the speed of convergence.

BRIEF DESCRIPTION OF DRAWINGS

In the accompanying drawings like reference numbers denote like components, brief description of each of which is herewith given.

Figure 1 shows a general structural block diagram of the main embodiment of the invention,

Figure 2 illustrates, by way of example, the performance of the present invention in extracting a sinusoidal signal,

Figure 3 illustrates, by way of example, the performance of the present invention in tracking a step change in the amplitude of the input signal,

Figure 4 illustrates, by way of example, the performance of the present invention in tracking a step change in the frequency of the input signal,

Figure 5 illustrates, by way of example, the effect of the presence of noise,

Figure 6 depicts a functional block diagram of the present invention,

Figure 7 presents the functionality of the present invention as a PLL,

Figure 8 illustrates the use of a filter in the main embodiment of the present invention to enhance its performance,

Figure 9 depicts employment of the core structure of the present invention in parallel and cascade combinations, and finally

Figure 10 presents a configuration employing a number of main embodiment of the invention for the elimination of undesirable components.

BEST MODE FOR CARRYING OUT THE INVENTION

Let $u(t)$ represent a signal which is registered by a sensor to indicate a natural phenomenon or is artificially generated by the use of some equipment. It is usually taken to be a function of time t although it can equivalently be a function of any other variable, and may be of an electromagnetic nature, such as voltage or current signals in electrical systems, or of a mechanical nature such as sound waves. Very often, signals exhibit some periodicity in which case they can be represented as a series of individual sinusoidal components. If a signal is not perfectly periodic and its characteristics happen to change over time, so will the parameters of constituting sinusoidal components. In such cases, it is desirable to extract individual constituting sinusoidal components and track their variations over time.

With reference to the accompanying drawings and in particular with reference to Figure 1, the present method of extraction of individual constituting sinusoidal components of a given signal comprises a number of simple steps to provide estimated values of the amplitude, phase and frequency of a more or less specified component of a given signal as well as the synthesized desired component itself.

The mechanism of the present method will now be explained in detail. A given input signal $u(t)$ is used to obtain an output signal $y(t)$ which is the desired constituting component of such a given signal. Such a synthesized output signal is subtracted from the given input signal by means of a subtraction operation 5. The outcome of such an operation is another signal $e(t)$ which, by the very fact that it is the difference between the given input signal and the synthesized desired component, is the totality

of the undesirable components and the error incurred in the extraction process. This operation can be concisely formulated by the following equation:

$$e(t) = u(t) - y(t). \tag{1}$$

According to the method of the invention, the instantaneous rate of change, or equivalently time derivative, of the amplitude of the desired component of the signal is taken to be proportional to the error signal $e(t)$ and the sine of the total estimated phase of the desired component, and hence it is proportional to their product. Therefore, a multiplication operation 10, provides the product of the error signal $e(t)$ and the sine of the total estimated phase of the desired component. Thus, a proportionality constant μ_1 15, when multiplied by the outcome of the product operation 10, yields the instantaneous rate of change of the amplitude of the desired component which, when integrated by an integration operation 20, provides the estimated value of the amplitude of the desired component of the input signal. The value of the initial condition of the integration 20 can be taken as zero in which case the procedure outlined by the method of invention is initialized from zero amplitude. It is easy to formulate the procedure just outlined for the estimation of instantaneous value of the amplitude of the desired component of a given input signal by the following equation:

$$\dot{A} = \mu_1 e \sin\phi \tag{2}$$

where the dot on top denotes the time derivative, and A and ϕ denote the amplitude and phase, respectively.

Quite similarly, according to the method of the invention, the instantaneous rate of change of the frequency of the desired component of the input signal is taken to be proportional to the error signal $e(t)$, the estimated value of amplitude $A(t)$ and the cosine of the total estimated phase of the desired component, and hence it is proportional to their product. Therefore, a multiplication operation 30 provides the product of the error signal $e(t)$, the estimated value of the amplitude $A(t)$ and the cosine of the total estimated phase of the desired component. Thus, a proportionality constant μ_2 40, when multiplied by the outcome of the product operation 30, yields the instantaneous rate of change of the frequency of the desired component which, when integrated by an

integration operation 60, provides the estimated value of the frequency of the desired component of the input signal. The value of the initial condition of the integration 60 can either be taken as zero in which case the procedure outlined by the method of invention finds the sinusoidal component of the input signal whose frequency is closest to zero, or can be set to any desired initial frequency $f_o = \frac{\omega_o}{2\pi}$ in which case the procedure outlined by the method of invention finds the sinusoidal component of the input signal the frequency of which is closest to such a desired value f_o. This conveniently furnishes the procedure of the invention with a method of specifying the desired component of the input signal the extraction of which is sought. The extracted component of the signal will therefore be the one angular frequency of which is closest to the value of the initial condition ω_o 50. It is easy to formulate the procedure just outlined for the estimation of the instantaneous value of the frequency of the desired component by the following equation:

$$\dot{\omega} = \mu_2 e A \cos\phi \tag{3}$$

where ω represents the value of the angular frequency of the desired component of the input signal; frequency f in Hz is $f = \frac{\omega}{2\pi}$.

According to the method of the present invention, the instantaneous rate of change of the total phase of the desired component of the input signal is the sum of the angular frequency ω itself and a constant factor of its instantaneous rate of change. This can be formulated as

$$\dot{\phi} = \omega + \mu_3 \dot{\omega}, \tag{4}$$

or with reference to the procedure of obtaining the estimation of frequency formulated by equation numbered (3), as

$$\dot{\phi} = \omega + \mu_2 \mu_3 e A \cos\phi.$$

Thus, a combined constant factor of $\mu_2\mu_3$ 35 provides the scaled product which is then added to the angular frequency by means of an addition operation 45 to yield the instantaneous rate of change of the total phase of the desired component of the signal which, when is integrated by integration 55, yields the estimated value of the phase of the desired component of the input signal. The value of the initial condition of the integration 55 can be taken as zero in which case the procedure outlined by the method

of invention is initialized from zero phase. The sine function 65 and the cosine function 70 generate the sine and cosine of thus estimated total phase. The function of the sine operation 65 and cosine operation 70 can be interchanged without any effect on the performance since this would only mean a different initial condition of integration 55.

The desired component of the input signal is a single sinusoid having an amplitude of $A(t)$ and a phase of $\phi(t)$ generated as outlined by the method of invention and can therefore be generated by multiplying the amplitude and the sine of the phase of the desired component by a product operation 25. This operation may be formulated as

$$y(t) = A\sin\phi. \tag{5}$$

Equations (1) to (5) summarize the steps of the procedure outlined by the method of the present invention. When put together, the equations outlining the method of invention are

$$\begin{aligned}
\dot{A} &= \mu_1 e \sin\phi, \\
\dot{\omega} &= \mu_2 e A \cos\phi, \\
\dot{\phi} &= \omega + \mu_3\dot{\omega}, \\
y(t) &= A\sin\phi, \\
e(t) &= u(t) - y(t).
\end{aligned}$$

It has been observed that the nonlinear non-autonomous dynamical system represented by the above set of differential equations possesses a unique asymptotically stable periodic orbit which lies in a neighborhood of the orbit associated with the desired component of the function $u(t)$. In terms of the engineering performance of the system, this indicates that the output of the system $y(t) = A\sin\phi$ will approach a sinusoidal component of the input signal $u(t)$. Moreover, the slow variations of the parameters in $u(t)$ are tolerated by the system.

The convergence speed in tracking variations in the amplitude of the input signal is observed to be mainly controllable by the assignment of the value of the parameter

μ_1. The larger μ_1 is chosen, the faster the embodiment of the method of invention follows variations in amplitude. The inherent trade-off existing in tracking capability is with accuracy. The larger μ_1 is chosen, the higher level of error is introduced in the convergence mechanism of the method of the invention. Likewise, the assignment of values of parameters μ_2 and μ_3 provides a means of controlling phase-frequency tracking speed versus accuracy of the method of invention. Roughly speaking, the larger the value of μ_2 is assigned, the faster the embodiment of the method of invention follows variations in the phase and a higher convergence error occurs.

One way of improving the aforementioned speed/error trade-off is the introduction of low pass filters before or after, or equivalently within, the integration operations 20 and 60. This allows for the assignment of large values of parameters μ_1 and μ_2 while maintaining the error within a given desired range.

Figure 1, while outlining the detailed steps of the procedure of the method of the present invention, shows the implementation of a system based on the method of the invention in the form of the composition of simple blocks, and therefore presents a system for the extraction of sinusoids of time-varying nature. Such an apparatus may be implemented using digital hardware in which case digital circuits are used to perform the required arithmetic operations. An example of such an embodiment of the present invention would be a system implemented on a field programmable gate array (FPGA) platform.

Alternatively, analog circuitry may be employed to construct the blocks of an apparatus devised on the basis of the method of the present invention as shown in Figure 1.

Other arrangements, such as one employing mechanical components to perform arithmetic tasks, may be constructed based on the method of the present invention.

Yet alternatively, the procedure outlined by the method of the invention can be easily implemented numerically within a software environment. Such a software code may then be executed by the use of computers, microprocessors, microcontrollers or digital

signal processor (DSP) platforms or other computational devices. Numerically, one, and not the only, possible way of expressing the set of equations governing the method of the invention in discrete form is

$$
\begin{aligned}
A[n+1] &= A[n] + \mu_1 T_s e[n] \sin\phi[n], & (6)\\
\omega[n+1] &= \omega[n] + \mu_2 T_s e[n] A[n] \cos\phi[n], & (7)\\
\phi[n+1] &= \phi[n] + T_s\omega[n] + \mu_2\mu_3 T_s e[n] A[n] \cos\phi[n], & (8)\\
y[n] &= A[n]\sin\phi[n], & (9)\\
e[n] &= u[n] - y[n]. & (10)
\end{aligned}
$$

where a first order approximation for the derivatives is assumed, T_s is the sampling time and n is the index of iteration.

The outlined method of signal analysis and synthesis is general in the sense that it offers a signal processing tool capable of extracting a desired sinusoid which may undergo variations in all three parameters amplitude, phase and frequency. Very often, the component of interest is specified in terms of its frequency implying a priori that the frequency is more or less known and almost fixed. In such situations, the procedure of the method of invention and thus the structure of the system implemented on that basis can be a bit further simplified. For this matter, supposing that the frequency is more or less fixed around $\omega_o = 2\pi f_o$, one can rewrite the equations summarizing the method of invention and obtain

$$
\begin{aligned}
\dot{A} &= \mu_1 e \sin\phi,\\
\dot{\phi} &= \omega_o + \mu_2 e A \cos\phi,\\
y(t) &= A\sin\phi,\\
e(t) &= u(t) - y(t).
\end{aligned}
$$

It is noteworthy that here the parameter μ_2 is reused and is equal to $\mu_2\mu_3$ 35 of Figure 1.

Another special case is when the parameter μ_3 is taken to be zero, yielding the following set of equations:

$$\begin{aligned}
\dot{A} &= \mu_1 e \sin\phi, \\
\dot{\omega} &= \mu_2 e A \cos\phi, \\
\dot{\phi} &= \omega, \\
y(t) &= A\sin\phi, \\
e(t) &= u(t) - y(t).
\end{aligned}$$

The procedure outlined by these equations and the system constructed thus are observed to perform well under more or less fixed frequency condition. This is similar to the first special case in terms of performance. It is however not as simple; the compensating advantage is that like the general case a direct estimation of the frequency is available; an estimation which is more or less a constant quantity.

The two special cases just mentioned do not have to be classified separately as they automatically result from the general case under specific conditions. Another special case arises when one eliminates the amplitude term in equation (3) of the set of equations summarizing procedure of the method of invention. This special case is of particular importance in the sense that it does not automatically result from the general case and has to be considered separately. In this case, the equations may be written as

$$\begin{aligned}
\dot{A} &= \mu_1 e \sin\phi, \\
\dot{\omega} &= \mu_2 e \cos\phi, \\
\dot{\phi} &= \omega + \mu_3 \dot{\omega}, \\
y(t) &= A\sin\phi, \\
e(t) &= u(t) - y(t).
\end{aligned}$$

This alternative procedure and the system implemented thereupon are also observed to perform well under all possible conditions.

The procedure outlined by the method of invention, mathematically described by the equations throughout this disclosure, refers to a dynamical system which may be described in many other ways. For example, it is known in the art that a given set of equations can be converted to an alternative set of equations by an act of change of

variables. For instance, equations (1) to (5), or those describing special cases of the latter, are framed in cylindrical coordinate system having dimensions (A, ϕ, ω). Obviously, a change of variables may be employed to frame the same equations in Cartesian coordinates system. As another example, it is known in the art that the parameters μ_1, μ_2 and μ_3, being arbitrary positive numbers yet to be determined according to a specific application, may be replaced by other positive quantities such as $\mu_1 A^{2k}$, $\mu_2 A^{2k}$ and $\mu_3 A^{2k}$, A and k being the amplitude and an arbitrary integer, respectively. Variations of this sort, exemplified by the two change of variables just mentioned, although result in equations of different appearance, are essentially the same as the general case described in this disclosure and are not considered separately.

INDUSTRIAL APPLICABILITY

The dynamics of the algorithm offered by the method of the present invention presents a notch filter in the sense that it extracts (i.e. lets pass through) one specific sinusoidal component and rejects all other components including noise. It is adaptive in the sense that the notch filter accommodates variations of the characteristics of the desired output over time.

The analogy with notch filters should not limit the applicability of the method of the invention. From a different perspective, the algorithm of the invention, or a number of such core algorithms employed in parallel, can be thought of as a frequency domain analysis tool such as Fourier transform. From an entirely different perspective, the present technique presents a new PLL structure.

To illustrate the performance of the present invention in its general form, a number of simulations are now presented. Figure 2 shows the extracted signal of an input of unit amplitude pure sinusoid with frequency $f = 60$ Hz and random phase. The initial conditions are chosen as $f_o = 50$ Hz, $A_o = 0$, and $\phi = 0$. With a moderate set of parameters, $\mu_1 = 200$, $\mu_2 = 20000$, $\mu_3 = 0.02$, the convergence is achieved in a few cycles.

Figure 3 shows the performance of the present invention in tracking time variations in the amplitude. A step change of 10% in the amplitude of the input signal, which is otherwise taken to be the same as before, occurs. The new value of the amplitude is tracked in a few cycles while the phase and frequency of the signal undergo transients for a few cycles.

Figure 4 shows a similar phenomenon in which the frequency of the input signal undergoes a step change. It is observed that the variations are effectively tracked with a transient of a few cycles. Values of the parameters are retained the same as before.

As far as the robustness of the system of the invention with regard to its internal structure is concerned, and most importantly with regard to the adjustment of parameters, it has been observed that performance of the system of the invention is almost unaffected by parameter variations of as large as 50%. The embodiment of the invention is found extraordinarily insensitive to variations of even of orders of magnitude in its internal setting. This merit is well appreciated in the context of some methods in which variations of the order of 0.1% may pose serious stability or performance problems.

In noisy environments, one cannot achieve the maximum speed of convergence without a trade-off with accuracy. Figure 5 shows the performance of the present invention in noisy environments. It is the same scenario as that of Figure 2 with the only difference that a white noise at the 20 dB below the level of sinusoidal input (i.e. of 10% of the magnitude) is added to the input signal. The extracted frequency is picked as an index of performance. It is found that the presence of 10% noise in the input generates an error of about 0.5% in the frequency estimation at the same speed of convergence. It is notable that for each given application, one can modify the values of μ-parameters to accordingly balance the speed and accuracy.

The procedure outlined by the method of the invention and the structure of the system constructed thereupon are extremely simple. To elaborate further on this fact, a comparison is now made between the present invention and the discrete Fourier transform (DFT). DFT is well-known for its simplicity of structure and hence its short execution

time or little computational demand if implemented numerically in a software environment. Using DFT, if written out in recursive form, the real and imaginary parts of the fundamental phasor, for example, of the input signal $u(t)$ can be iteratively computed as

$$a_1[n] = a_1[n-1] + \frac{2}{N}(u[n] - u[n-N])\cos(\frac{2\pi n}{N}), \tag{11}$$

$$b_1[n] = b_1[n-1] + \frac{2}{N}(u[n] - u[n-N])\sin(\frac{2\pi n}{N}) \tag{12}$$

where $u[n-N]$ is the input sample corresponding to the previous period which is saved in memory. The fundamental component of $u(t)$ is expressed as

$$u_1(t) = A_1 \sin(\omega_o t + \theta_1) \tag{13}$$

in which A_1 and θ_1 can be calculated from (11) and (12) by

$$A_1 = \sqrt{a_1^2 + b_1^2}, \tag{14}$$

$$\theta_1 = \tan^{-1}(\frac{b_1}{a_1}). \tag{15}$$

The two recursive equations have very simple structure. Two multiplications are the major computations involved. However, as many samples as the window length should be saved in the memory. Note that a relatively large amount of computation is required to provide a signal $u_1(t)$ out of a_1 and b_1. A square root and an arctangent operation are to be performed. These two functions are not straightforward to implement and need a considerable amount of memory and hardware. Moreover, further work has to be done to synthesize a synchronous signal $u_1(t)$, using a PLL for example.

Since the method of the present invention is not window-based, no input data is required to be saved in the memory. It only needs to have the instantaneous value of the input. To compare the method of invention with DFT from the computational order point of view, one needs to consider all the required computations by both methods to provide the same result. DFT needs to do all the computations in (11) to (15) to provide a fundamental, its amplitude, and its phase angle. In the procedure outlined by the method of the present invention it is needed to perform the computations in the set of equations (6) to (10) to provide the same results. It is obvious that the procedure

outlined by the method of the present invention lends itself more easily than DFT to the practical implementation in terms of computational volume of the software. From a hardware point of view, since the extracted fundamental and distortion are directly available, there is no need for any synchronizing tool such as PLL in the method of the present invention.

From one perspective, the present invention is an adaptive amplitude, phase and frequency estimator. This functional description of the invention is pictured in Figure 6. In this sense, it belongs to the general category of signal estimation systems and methods such as those widely employed in phasor measurement. It can also be thought of as an adaptive notch filter whose center frequency is adjustable and is flexibly moving. In this sense, it is comparable with adaptive filters. It can be envisaged as a signal analysis tool which analyzes individual components of frequency decomposition of a given signal. In this sense, it is comparable with DFT.

Unlike most signal processing techniques which provide estimates of characteristics of the components of a given signal, the present invention not only does provide estimates of the amplitude, total or constant phase and frequency, but also synthesizes the desired sinusoidal component itself in real time. This is feasible due to the fact that the total phase is estimated at each instant. From this perspective, the present invention is a new PLL as pictured in Figure 7.

The input signal, out of which the system is supposed to extract a clean single sinusoid, may contain components lying in frequency bands far off that of interest. A filter may be inserted at the input of the core unit to eliminate some of the undesired components and thus enhance the speed-accuracy trade-off of the unit. Obviously, this filter does not need to be sharp meaning that its structure is very simple and the phase delay will not be excessively long. The phase delay and the gain of the filter will be functions of frequency. The procedural algorithm of the method of invention, by the very fact that it synchronously estimates frequency, amplitude and total phase, allows for the correction of the phase delay and change in amplitude. The estimation of frequency at each instant may be used to obtain the phase delay caused by the filter. Such a quantity

is then subtracted from the total phase which is also instantly generated. Likewise, the amplitude can be corrected. The desired output can then be simply synthesized as illustrated in Figure 8. A particular example of application of the configuration of Figure 8 is a power signal interference eliminator. Such a device is comprised of a filter such as that shown in Figure 8 to extract the interfering power line signal which is then subtracted from the input to provide a clean signal out of which the power signal is removed.

As a signal analysis tool, a number of core units, or configuration of Figure 8 as core units for an improved performance, may be connected in parallel, or cascaded, to decompose the multi-component input signal into its constituting sinusoidal components. Figure 9 shows two possible combinations. Of course, one can conceive of other combinations to build multi-core units tailored by needs of any given application. For example, if the desired component is masked by components of a more or less known frequency composition, undesirable components may be extracted and subtracted from the input signal first to provide a cleaner input for the unit which is supposed to extract the desired component. Figure 10 shows such a configuration. It is noteworthy that each core tries to extract a sinusoidal component the frequency of which is closest to its specified initial condition. In order to avoid overlapping duties of the core units, the operating frequency of each of the units has to be within a specified range. This can be achieved by means of introducing range limiters within frequency integration 60 of Figure 1, for instance.

WHAT IS CLAIMED IS:

1. A method of extraction of at least one sinusoidal component of an input signal, comprising the steps of
estimation of the amplitude of said sinusoidal component of said input signal;
estimation of the frequency of said sinusoidal component of said input signal;
estimation of the total phase of said sinusoidal component of said input signal;
synthesization of said sinusoidal component of said input signal; and
estimation of an error signal representing the difference between said input signal and said sinusoidal component of said input signal by the use of a first subtraction,
wherein:
synthesized sinusoidal component of said input signal is the desired component of said input signal to which convergence is sought and specification of which is accomplished by predetermination of its frequency, and
said error signal is the totality of the undesired components present in said input signal, including noise, compounded by the incurred extraction error.

2. A method of extraction of sinusoids as defined in claim 1 wherein said step of estimation of the amplitude comprises a first integration of a first product of said error signal and a sine, or a cosine, of the total phase of said sinusoidal component of said input signal, scaled by a first scaling factor,
wherein:
the value of the initial condition of said first integration is a real number; and
said first scaling factor is a positive number which primarily determines the amplitude-tracking speed of the sinusoid extraction method defined in accordance with claim 1.

3. A method of extraction of sinusoids as defined in claim 1 wherein said step of estimation of frequency comprises a second integration of a second product of said error signal, the amplitude of said sinusoidal component of said input signal, and a cosine, or a sine, of the total phase of said sinusoidal component of said input signal, scaled by a second scaling factor,

wherein:

the value of the initial condition of said second integration is a positive number, assignment of which provides a way of predetermination of desired component of said input signal; and

said second scaling factor is a positive number which partially determines the phase-frequency-tracking speed of the sinusoid extraction method defined in accordance with claim 1.

4. A method of extraction of sinusoids as defined in claim 1 wherein said step of estimation of the total phase comprises a third integration of a summation of frequency of said sinusoidal component of said input signal and scaled by a third scaling factor of the time-derivative of the frequency of said sinusoidal component of said input signal, wherein:

the value of the initial condition of said third integration is a real number, and

said third scaling factor is a positive number, or zero, which partially determines the phase-frequency-tracking speed of the sinusoid extraction method defined in accordance with claim 1.

5. A method of extraction of sinusoids as defined in claim 1 wherein said step of synthesization of said sinusoidal component of said input signal comprises a third product of said sine, or said cosine, of the total phase of said sinusoidal component of said input signal and the amplitude of said sinusoidal component of said input signal.

6. A method of extraction of sinusoids as defined in claim 1 wherein said step of estimation of the amplitude as defined in accordance with claim 2 further comprises the refinement of the estimated value of the amplitude of said sinusoidal component of said input signal by using a first low pass filtering within said first integration.

7. A method of extraction of sinusoids as defined in claim 1 wherein said step of estimation of the frequency as defined in accordance with claim 3 further comprises the refinement of the estimated value of the frequency of said sinusoidal component of said input signal by using a second low pass filtering within said second integration.

8. A method of extraction of at least one sinusoidal component of an input signal, comprising the steps of

filtering said input signal to generate a filtered input signal;

extracting said sinusoidal component of said filtered input signal and its amplitude, phase and frequency according to the method defined in claim 1;

correcting amplitude and phase of said sinusoidal component of said filtered input signal; and

synthesizing said sinusoidal component of said filtered input signal,

wherein:

frequency of said sinusoidal component of said filtered input signal is used in said step of correcting the amplitude and phase.

9. A method of extraction of sinusoids as defined in claim 1 wherein said second product defined in accordance with claim 3 is the product of said error signal and a cosine, or a sine, of the total phase of said sinusoidal component of said input signal.

10. A method of elimination of at least one sinusoidal component of an input signal comprising the steps of

extraction of said sinusoidal component of said input signal according to the method defined in claim 1; and

subtraction of said sinusoidal component of said input signal from said input signal by the use of a second subtraction.

11. A method of extraction of at least one sinusoidal component of an input signal comprising the steps of

elimination of at least one undesired sinusoidal component of an input signal according to the method defined in claim 10 to generate an artifact-free input signal; and

extraction of said sinusoidal component of said artifact-free input signal according to the method defined in claim 1.

12. A method of extraction of a plurality of sinusoidal components of an input signal comprising multiple steps of extraction of a sinusoidal component of an input signal

according to the method defined in claim 1 wherein frequency range within said second integration operation is confined within a pre-specified range.

13. A system for the extraction of at least one sinusoidal component of an input signal, comprising the means of

a) estimation of the amplitude of said sinusoidal component of said input signal by means of a first integration of a first product of an error signal and a sine, or a cosine, of the total phase of said sinusoidal component of said input signal, scaled by a first scaling factor,

b) estimation of the frequency of said sinusoidal component of said input signal by means of a second integration of a second product of said error signal, the amplitude of said sinusoidal component of said input signal, and a cosine, or a sine, of the total phase of said sinusoidal component of said input signal, scaled by a second scaling factor;

c) estimation of the total phase of said sinusoidal component of said input signal by means of a third integration of a summation of the frequency of said sinusoidal component of said input signal and scaled by a third scaling factor of time-derivative of the frequency of said sinusoidal component of said input signal;

d) synthesization of said sinusoidal component of said input signal by means of a third product of said sine, or said cosine, of the total phase of said sinusoidal component of said input signal and the amplitude of said sinusoidal component of said input signal; and

e) estimation of said error signal representing the difference between said input signal and said sinusoidal component of said input signal by means of a first subtraction.

14. A system for the extraction of sinusoids as defined in claim 13 wherein said means of estimation of amplitude further comprises the refinement of the estimated value of the amplitude of said sinusoidal component of said input signal by means of a first low pass filter incorporated within said first integration.

15. A system for the extraction of sinusoids as defined in claim 13 wherein said means of estimation of the frequency further comprises the refinement of the estimated value

of the frequency of said sinusoidal component of said input signal by means of a second low pass filter incorporated within said second integration.

16. A system for the extraction of at least one sinusoidal component of an input signal, comprising the means of

filtering said input signal to generate a filtered input signal;

extracting said sinusoidal component of said filtered input signal and its amplitude, phase and frequency according to the system defined in claim 13;

correcting the amplitude and phase of said sinusoidal component of said filtered input signal; and

synthesizing said sinusoidal component of said filtered input signal,

wherein:

the frequency of said sinusoidal component of said filtered input signal is used in said means of correcting the amplitude and phase.

17. A system for the extraction of sinusoids as defined in claim 13 wherein said second product is the product of said error signal and a cosine, or a sine, of the total phase of said sinusoidal component of said input signal.

18. A system of extraction of sinusoids as defined in claim 13 wherein at least one of said first or second subtraction, said first, second or third integration, said first, second or third product, said first, second or third scaling, said summation, said sine or said cosine operations is realized by the means of analog circuitry.

19. A system of extraction of sinusoids as defined in claim 13 wherein at least one of said first or second subtraction, said first, second or third integration, said first, second or third product, said first, second or third scaling, said summation, said sine or said cosine operations is realized by the means of digital circuitry.

20. A system of extraction of sinusoids as defined in claim 13 wherein at least one of said first or second subtraction, said first, second or third integration, said first, second or third product, said first, second or third scaling, said summation, said sine or said

cosine operations is realized by the means of a software program.

21. A system for the elimination of at least one sinusoidal component of an input signal comprising the means of
extraction of said sinusoidal component of said input signal according to the system defined in claim 13; and
subtraction of said sinusoidal component of said input signal from said input signal by means of a second subtraction.

22. A system for the extraction of at least one sinusoidal component of an input signal comprising the means of
elimination of at least one undesired sinusoidal component of an input signal according to the system defined in claim 21 to generate an artifact-free input signal; and
extraction of said sinusoidal component of said artifact-free input signal according to the system defined in claim 13.

23. A system for the extraction of a plurality of sinusoidal components of an input signal comprising means of multiple extraction of sinusoidal components of an input signal according to the system defined in claim 13 wherein the frequency range within said second integration operation is confined within a pre-specified range.

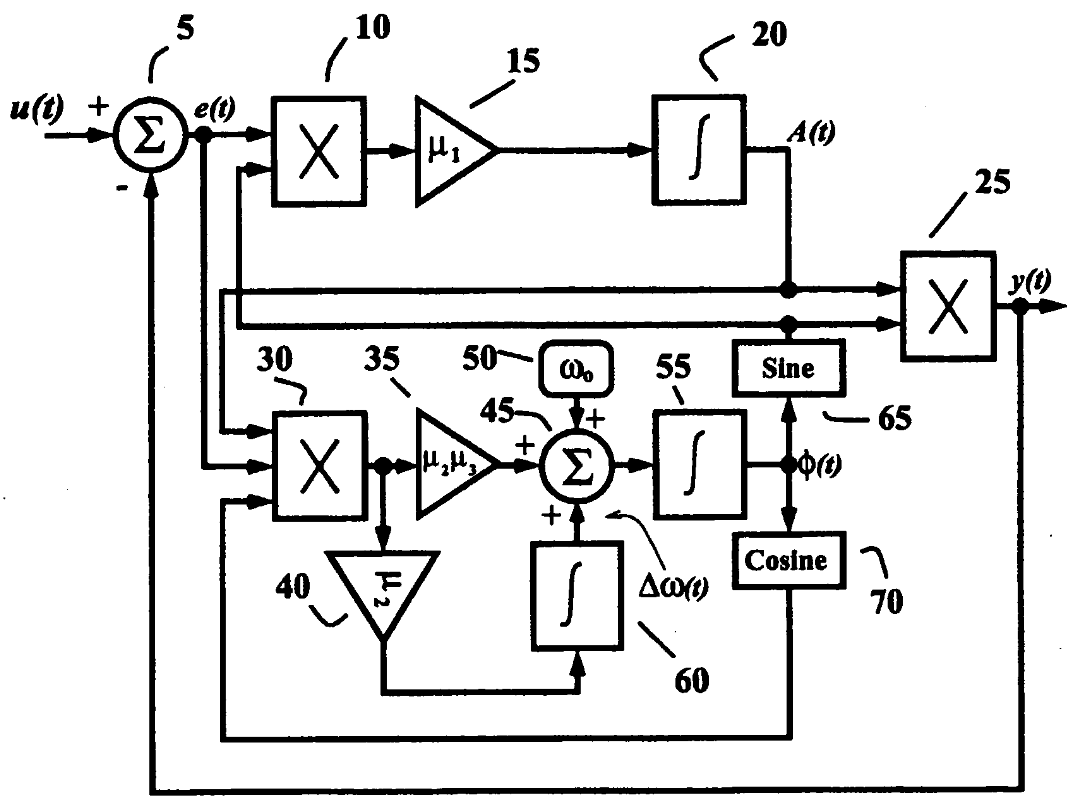
5
10
15
20
25
u(t)
+
-
e(t)
A(t)
y(t)
30
35
50
55
45
+
Sine
65
ϕ(t)
Cosine
70
40
Δω(t)
60
μ1
μ2μ3
μ2
ω0

FIG. 1

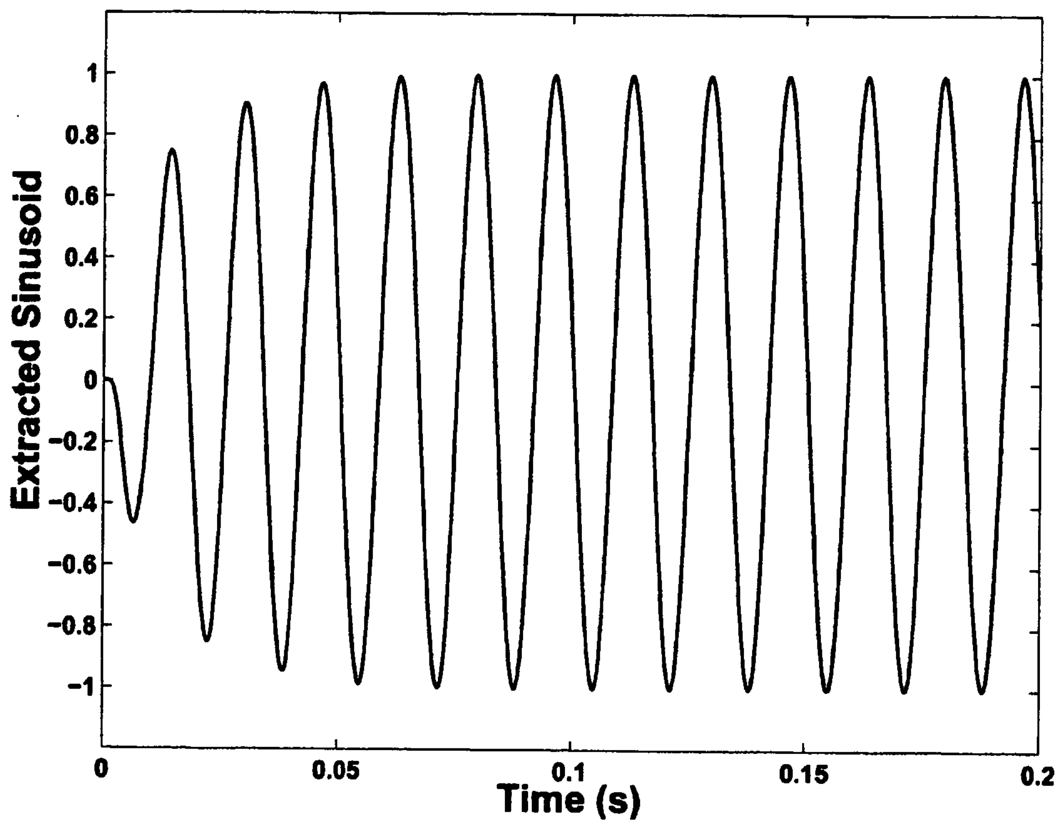
Extracted Sinusoid
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.05
0.1
0.15
0.2
Time (s)

FIG. 2

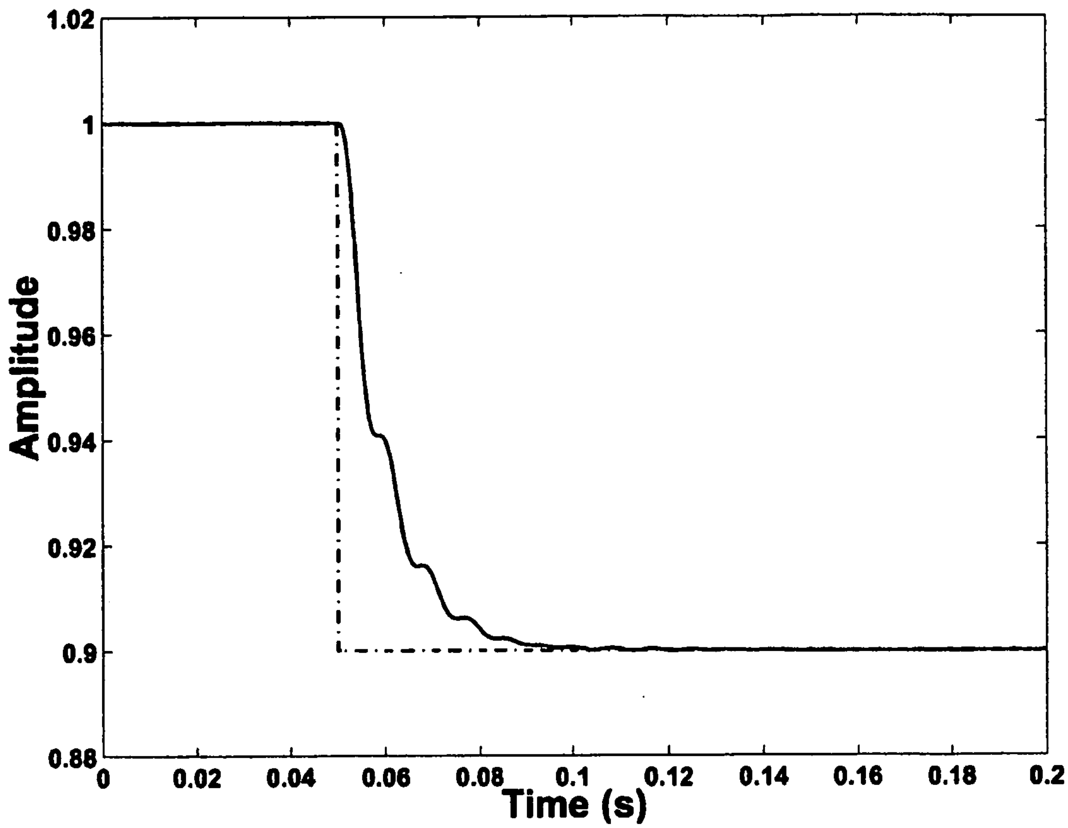
1.02
1
0.98
0.96
0.94
0.92
0.9
0.88
Amplitude
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
Time (s)

FIG. 3

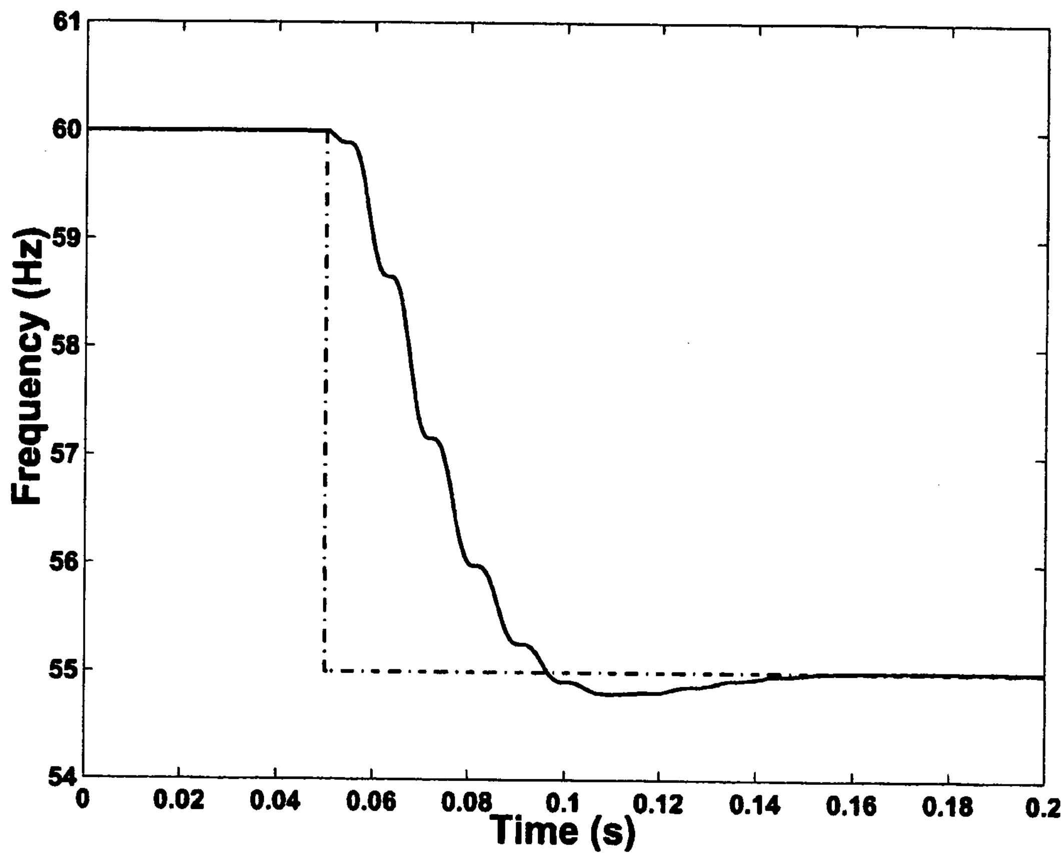
Frequency (Hz)
61
60
59
58
57
56
55
54
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
Time (s)

FIG. 4

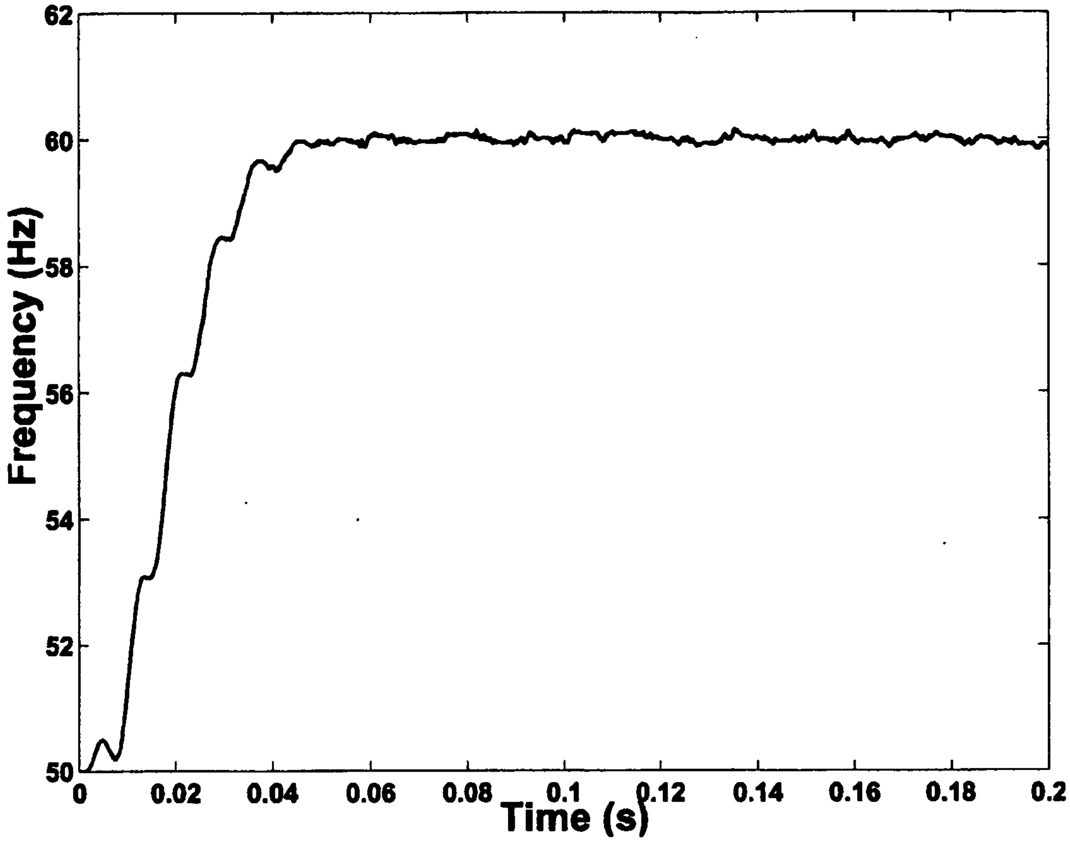
62
60
58
56
54
52
50
Frequency (Hz)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
Time (s)

FIG. 5

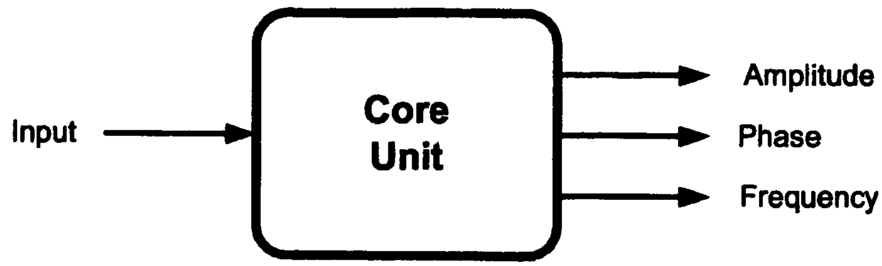
Input
Core
Unit
Amplitude
Phase
Frequency

FIG. 6

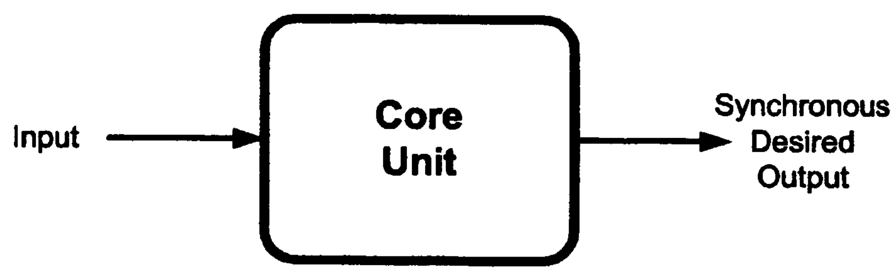
Input
Core
Unit
Synchronous
Desired
Output

FIG. 7

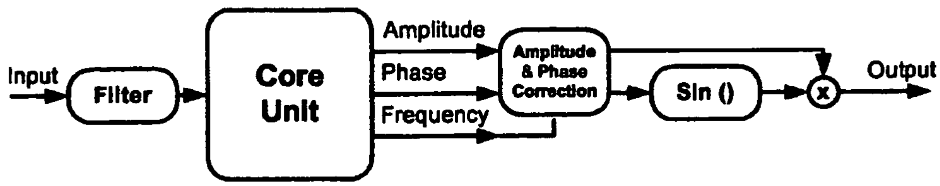
Input
Filter
Core Unit
Amplitude
Phase
Frequency
Amplitude & Phase Correction
Sin ()
x
Output

FIG. 8

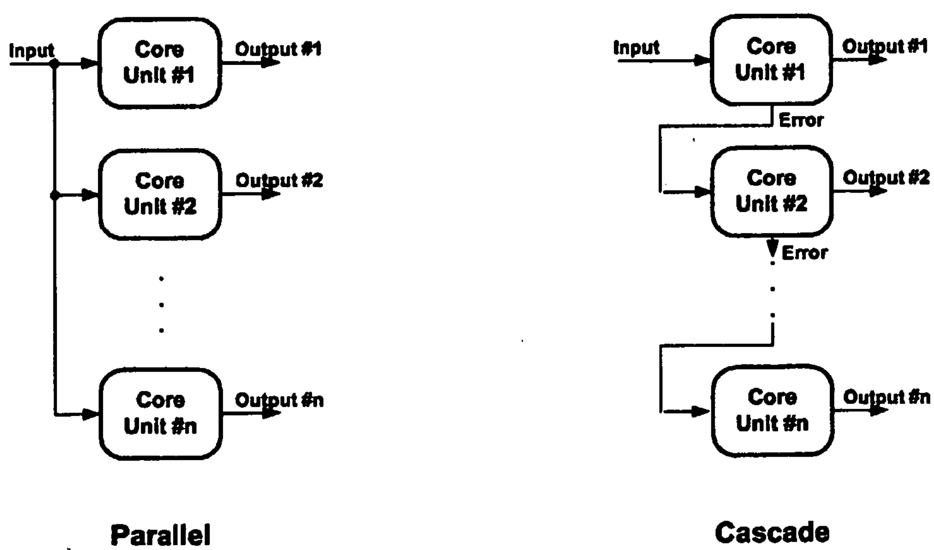
Input
Core Unit #1
Output #1
Core Unit #2
Output #2
Core Unit #n
Output #n
Parallel
Input
Core Unit #1
Output #1
Error
Core Unit #2
Output #2
Error
Core Unit #n
Output #n
Cascade

FIG. 9

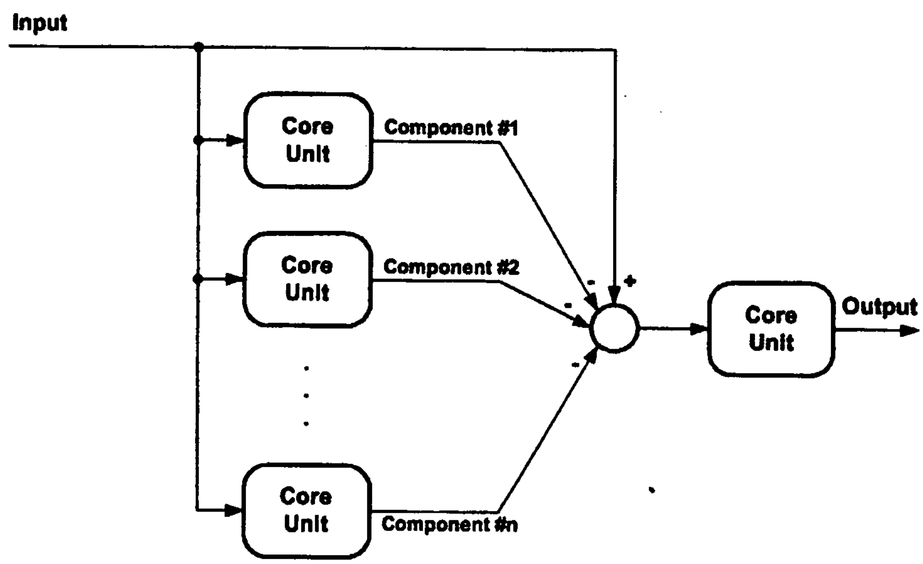
Input
Core Unit
Component #1
Core Unit
Component #2
Core Unit
Component #n
-
-
-
+
Core Unit
Output

FIG. 10